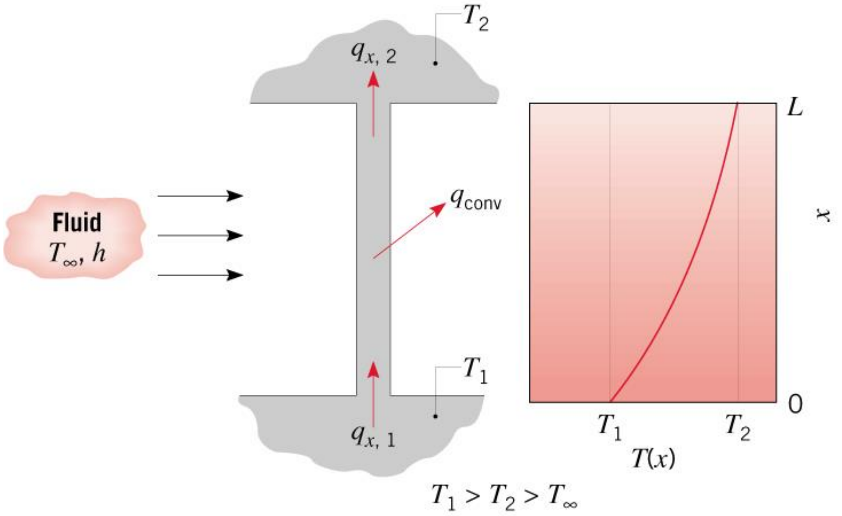
Extended Surfaces

Chapter Three

Section 3.6

Nature and Rationale of Extended Surfaces

- An extended surface (also know as a combined conduction-convection system or a fin) is a solid within which heat transfer by conduction is *assumed* to be one dimensional, while heat is also transferred by convection (and/or radiation) from the surface in a direction transverse to that of conduction.



  - If heat is transferred from the surface to the fluid by convection, what surface condition is dictated by the conservation of energy requirement?
  - Why is heat transfer by conduction in the *x*-direction not, in fact, one-dimensional?

– What is the actual functional dependence of the temperature distribution in the solid?

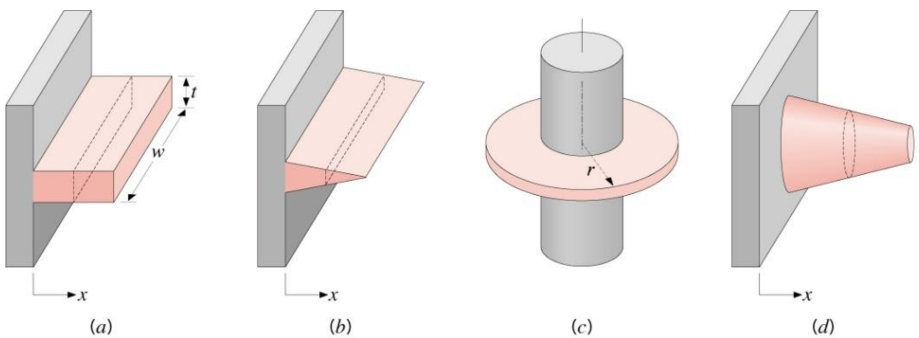
– If the temperature distribution is assumed to be one-dimensional, that is, $T=T(x)$, how should the value of T be interpreted for any x location?

– How does $q_{\text{cond},x}$ vary with $\boldsymbol{x}$?

– When may the assumption of one-dimensional conduction be viewed as an excellent approximation? The thin-fin approximation.

- Extended surfaces may exist in many situations but are commonly used as fins to enhance heat transfer by increasing the surface area available for convection (and/or radiation). They are particularly beneficial when h is small, as for a gas and natural convection.

- Some typical fin configurations:



Straight fins of (a) uniform and (b) non-uniform cross sections; (c) annular fin, and (d) pin fin of non-uniform cross section.

The Fin Equation

- Assuming one-dimensional, steady-state conduction in an extended surface of constant conductivity (k) and uniform cross-sectional area (A_c), with negligible generation $(\dot{q} = 0)$ and radiation $(q''_{\text{rad}} = 0)$, the *fin equation* is of the form:

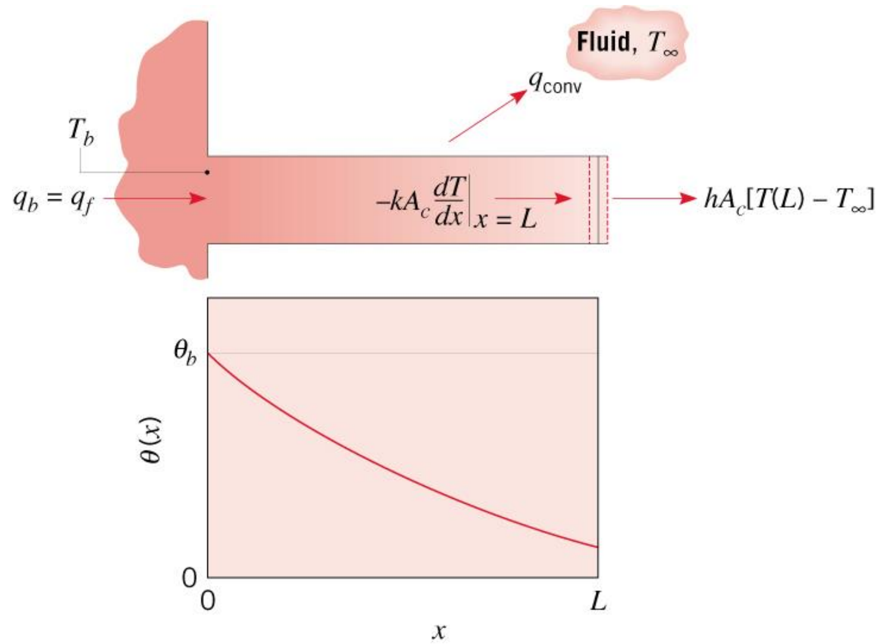
$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0 \qquad (3.67)$$

or, with $m^2 \equiv (hP / kA_c)$ and the reduced temperature $\theta \equiv T - T_\infty$,

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad (3.69)$$

How is the fin equation derived?

- Solutions (Table 3.4):



Base (x = 0) condition

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

Tip (x = L) conditions

A. Convection: $-kd\theta/dx|_{x=L} = h\theta(L)$

B. Adiabatic: $d\theta/dx|_{x=L} = 0$

C. Fixed temperature: $\theta(L) = \theta_L$

D. Infinite fin ($mL > 2.65$): $\theta(L) = 0$

- Fin Heat Rate:

$$q_f = -kA_c \frac{d\theta}{dx}\Big|_{x=0} = \int_{A_f} h\theta(x)\,dA_s$$

Fin Performance Parameters

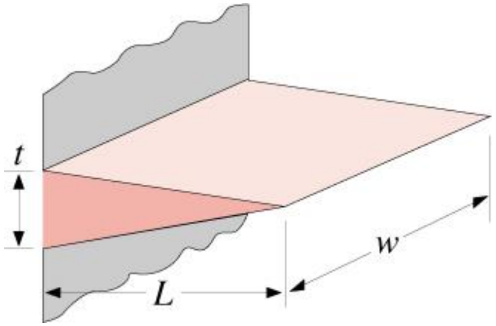
- Fin Efficiency:

$$\eta_f \equiv \frac{q_f}{q_{f,\max}} = \frac{q_f}{hA_f\theta_b} \qquad \text{where } 0 \le \eta_f \le 1 \qquad (3.91)$$

How is the efficiency affected by the thermal conductivity of the fin?

Expressions for η_f are provided in Table 3.5 for common geometries.

Consider a triangular fin:



$$A_f = 2w\left[L^2 + (t/2)^2\right]^{1/2}$$

$$A_p = (t/2)L$$

$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$$

- Fin Effectiveness:

$$\varepsilon_f \equiv \frac{q_f}{hA_{c,b}\theta_b} \qquad (3.86)$$

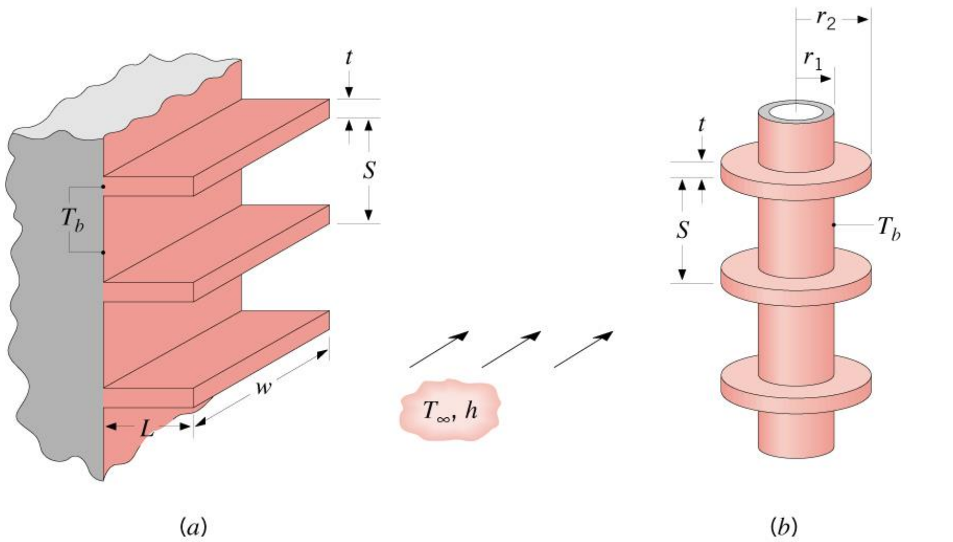
$$\varepsilon_f \uparrow \text{ with } \downarrow h, \uparrow k \text{ and } \downarrow A_c / P$$

- Fin Resistance:

$$R_{t,f} \equiv \frac{\theta_b}{q_f} = \frac{1}{hA_f\eta_f} \qquad (3.97)$$

Fin Arrays

- Representative arrays of
  (a) rectangular and
  (b) annular fins.



– Total surface area:

$$A_t = NA_f + A_b \quad (3.104)$$

Number of fins   Area of exposed base (*prime* surface)

– Total heat rate:

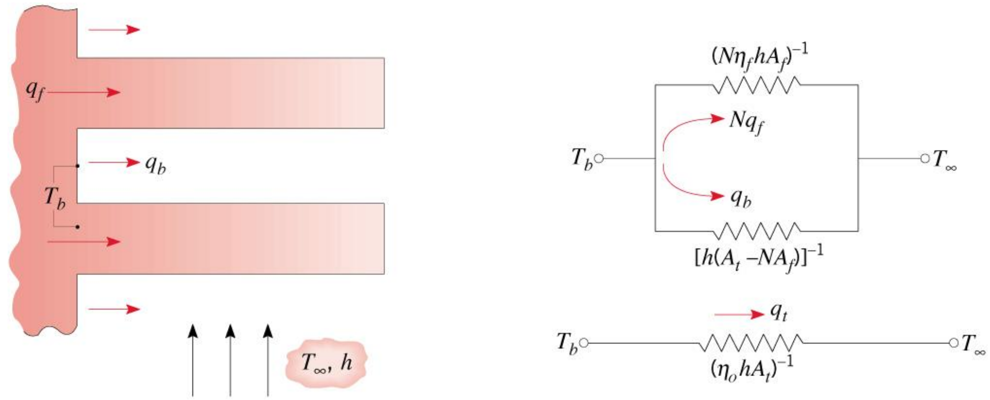
$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \equiv \eta_o hA_t\theta_b = \frac{\theta_b}{R_{t,o}} \quad (3.105)$$

– Overall surface efficiency and resistance:

$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1-\eta_f\right) \quad (3.107)$$

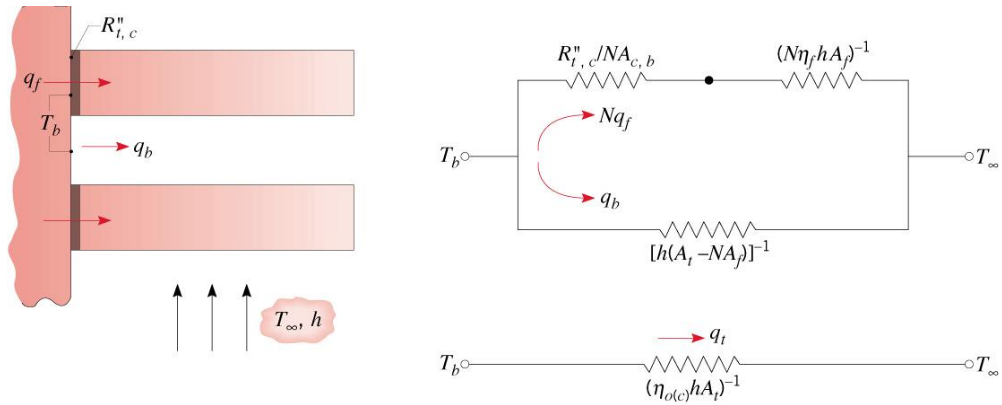
$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o hA_t} \quad (3.108)$$

- Equivalent Thermal Circuit:



(a)

- Effect of Surface Contact Resistance:



(b)

$$q_t = \eta_{o(c)} h A_t \theta_b = \frac{\theta_b}{R_{t,o(c)}}$$

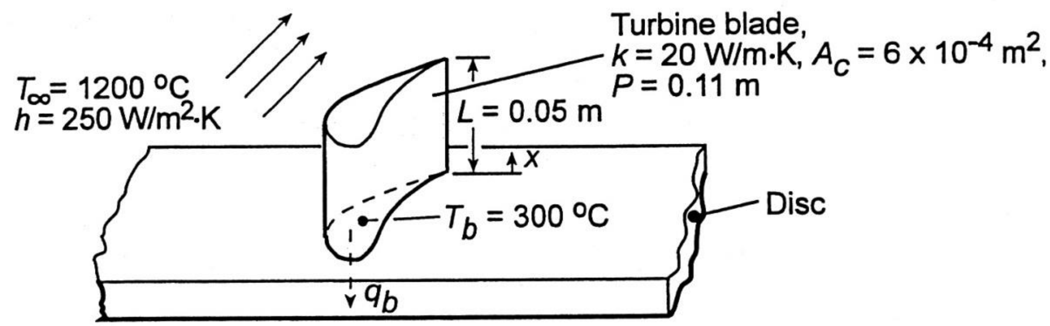
$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t}\left(1 - \frac{\eta_f}{C_1}\right) \qquad (3.110a)$$

$$C_1 = 1 + \eta_f h A_f \left(R''_{t,c} / A_{c,b}\right) \qquad (3.110b)$$

$$R_{t,o(c)} = \frac{1}{\eta_{o(c)} h A_t} \qquad (3.109)$$

Problem 3.99: Assessment of cooling scheme for gas turbine blade. Determination of whether blade temperatures are less than the maximum allowable value (1050°C) for prescribed operating conditions and evaluation of blade cooling rate.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant *k*, (3) Adiabatic blade tip, (4) Negligible radiation.

ANALYSIS: Conditions in the blade are determined by Case B (adiabatic tip) of Table 3.4.

(a) With the maximum temperature existing at $x = L$, Eq. 3.80 yields

$$\frac{T(L) - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh mL}$$

$$m = \left(hP/kA_c\right)^{1/2} = \left(250\ \text{W/m}^2 \cdot \text{K} \times 0.11\text{m}/20\text{W/m} \cdot \text{K} \times 6 \times 10^{-4}\text{m}^2\right)^{1/2} = 47.87\ \text{m}^{-1}$$

$$mL = 47.87\ \text{m}^{-1} \times 0.05\ \text{m} = 2.39$$

From Table B.1 (or by calculation), $\cosh mL = \cosh(2.39) = 5.51$. Hence,

$$T(L) = 1200^{\circ}\mathrm{C} + (300 - 1200)^{\circ}\mathrm{C}/5.51 = 1037^{\circ}\mathrm{C}$$

and, *subject to the assumption of an adiabatic tip*, the operating conditions are acceptable.

(b) With $M = \left(hPkA_c\right)^{1/2}\theta_b = \left(250\,\mathrm{W/m^2\cdot K} \times 0.11\,\mathrm{m} \times 20\,\mathrm{W/m\cdot K} \times 6\times10^{-4}\,\mathrm{m^2}\right)^{1/2}\left(-900^{\circ}\mathrm{C}\right) = -517\,\mathrm{W}$,

Eq. 3.81 and Table B.1 yield

$$q_f = M \tanh mL = -517\,\mathrm{W}(0.983) = -508\,\mathrm{W}$$

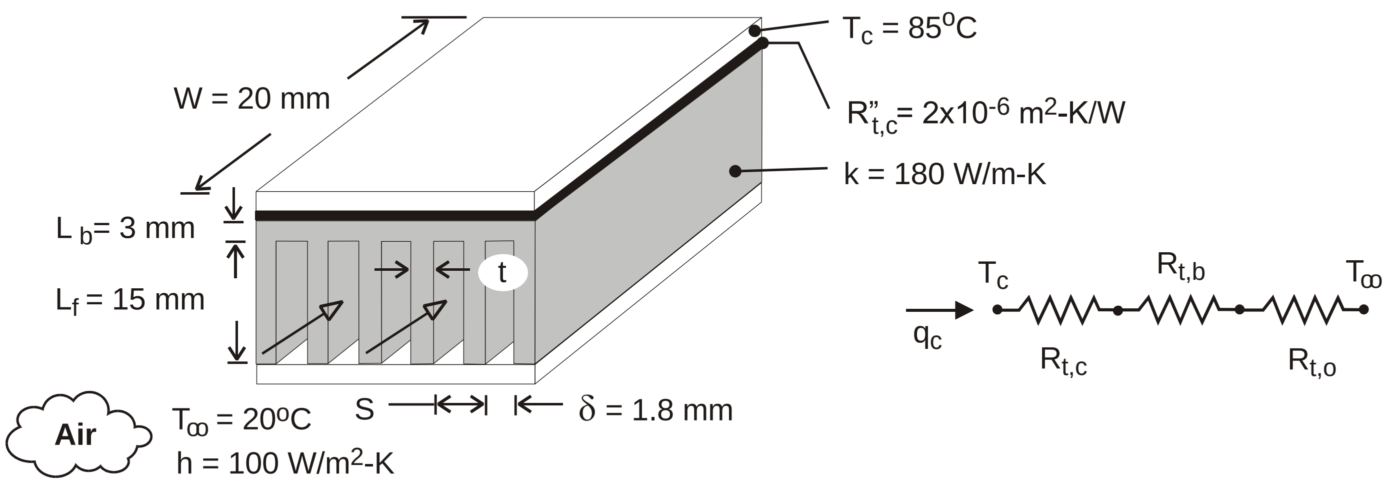
Hence,

$$q_b = -q_f = 508\,\mathrm{W}$$ <

COMMENTS: Heat transfer is to the base of the blade.
Radiation losses from the blade surface contribute to reducing the blade temperatures, but what is the effect of assuming an adiabatic tip condition? Calculate the tip temperature allowing for convection from the gas.

Problem 3.114: Determination of maximum allowable power q_c for a 20 mm × 20 mm electronic chip whose temperature is not to exceed $T_c = 85^\circ\mathrm{C}$, when the chip is attached to an air-cooled heat sink with N = 11 fins of prescribed dimensions.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surface of heat sink, (7) Negligible radiation, (8) Adiabatic fin tips.

ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{\text{tot}}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

$$R_{t,c} = R''_{t,c} / \text{W}^2 = 2 \times 10^{-6}\,\text{m}^2 \cdot \text{K/W} / \left(0.02\text{m}\right)^2 = 0.005\ \text{K/W}$$

$$R_{t,b} = L_b / k\left(\text{W}^2\right) \quad = 0.003\text{m} / 180\ \text{W/m} \cdot \text{K}\left(0.02\text{m}\right)^2 = 0.042\ \text{K/W}$$

From Eqs. (3.108), (3.107), and (3.104)

$$R_{t,o} = \frac{1}{\eta_o h A_t},\ \eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right)$$

$$A_f = 2WL_f = 2 \times 0.02\text{m} \times 0.015\text{m} = 6 \times 10^{-4}\ \text{m}^2$$

$$A_b = W^2 - N(tW) = (0.02\text{m})^2 - 11(0.182 \times 10^{-3}\ \text{m} \times 0.02\text{m}) = 3.6 \times 10^{-4}\ \text{m}^2$$

$$A_t = NA_f + A_b = 6.96 \times 10^{-3}\ \text{m}^2$$

With $mL_f = (2h/kt)^{1/2}\ L_f = (200\ \text{W/m}^2\cdot\text{K}/180\ \text{W/m}\cdot\text{K} \times 0.182 \times 10^{-3}\text{m})^{1/2}\ (0.015\text{m}) = 1.17$, $\tanh mL_f = 0.824$ and Eq. (3.94) yields

$$\eta_f = \frac{\tanh mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

$\eta_o = 0.719$,

$R_{t,o} = 2.00$ K/W, and

$$q_c = \frac{\left(85 - 20\right)°\text{C}}{\left(0.005 + 0.042 + 2.00\right)\text{K/W}} = 31.8\ \text{W}$$ <

contact base fin array

resistances

COMMENTS: The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with $h = 100\ \text{W/m}^2 \cdot \text{K}, R_{\text{tot}} = 2.05\ \text{K/W}$ from Part (a) would be replaced by $R_{\text{conv}} = 1/hW^2 =$ 25 K/W, yielding $q_c =$ 2.60 W.